

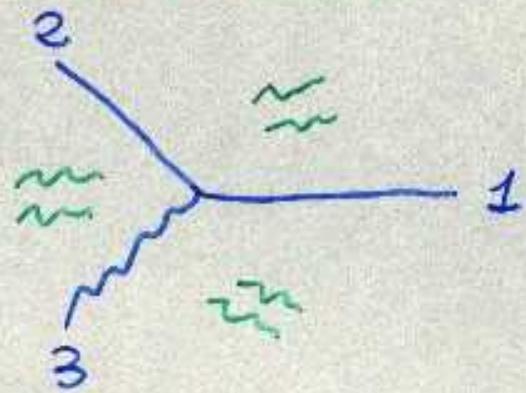
### 3 JET EVENTS: $K_{cut}$ DISTRIBUTION

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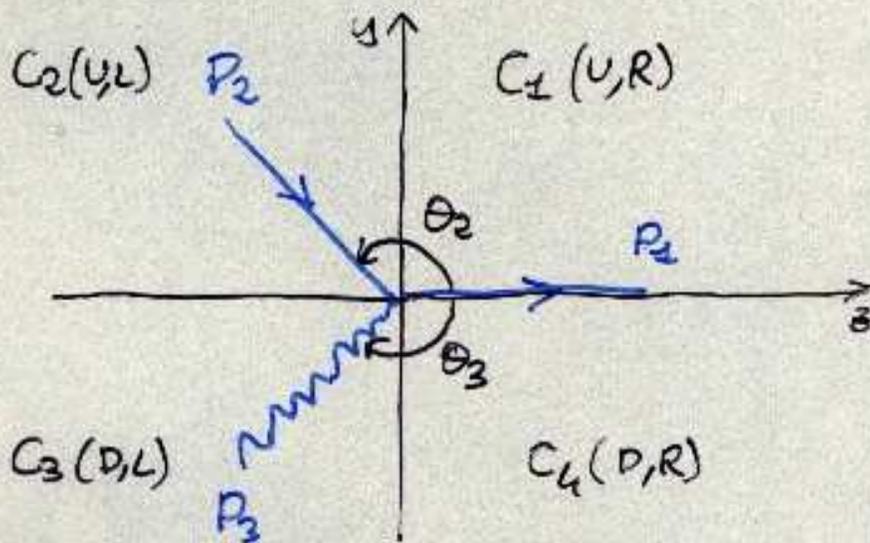
$$TQ = \max_b \sum_h P_b \cdot \eta = \sum_h |P_{h3}|$$

$$T_H Q = \max_{\eta \cdot \eta_{33} = 0} \sum_h P_b \cdot \eta = \sum_h |P_{h1}|$$

$$T_m Q = \sum_h |P_{hx}| \equiv K_{cut}$$



BORN LEVEL:  $K_{cut} = 0$



$$\underline{P}_2 = (0, 0, T \frac{Q}{2}) = \sum_{h \in R} \underline{P}_h \Rightarrow \sum_{h \in R} \underline{P}_{ht} = \sum_{h \in L} \underline{P}_{ht} = 0$$

$$\underline{P}_{2t} = (0, T_H \frac{Q}{2}) = \sum_{h \in U} \underline{P}_{ht} \Rightarrow \sum_{h \in U} P_{hx} = \sum_{h \in D} P_{hx} = 0$$

BEYOND BORN LEVEL:

'INTEGRATED'  $K_{cut}$  DISTRIBUTION  $T \sim T_H \gg T_m$

$$\frac{d\sigma}{dT dT_H} = \sum_m \int d\text{om} \Theta(K_{cut} - \sum_{h \in S} |P_{hx}|) \delta^3(\sum_{h \in R} \underline{P}_h - \underline{P}_2) \delta^3(\sum_{h \in U} \underline{P}_{ht} - \underline{P}_{2t})$$

$$DL \sim H_0^2(T, T_H) e^{-\frac{(2L+1) \frac{d\sigma}{\sigma} \log^2 \frac{Q}{K_{cut}}}{\pi}} \Sigma(K_{cut})$$

# SL CORRECTIONS

A. BANFI

- 1) HARD PARTON RECOIL EXPONENTIATION
- 2) ARGUMENT OF THE RUNNING COUPLING
- 3) HARD INTRA-JET PARTON DECAY
- 4) SOFT INTER-JET GLUON RADIATION

## 2 JET EVENTS

1) + 2) + 3)  $\Leftrightarrow$  COHERENT BRANCHING  
CORRECTIONS DEPEND ONLY ON THE NATURE  
OF HARD EMITTING PARTONS

## 3 JET EVENTS

ALSO 4)  $\Leftrightarrow$  QUANTUM INTERFERENCE  
CORRECTIONS ARE SENSITIVE TO 3 JET STRUCTURE



PREDICTIONS ARE  
GEOMETRY DEPENDENT!

A. B., G. Marchesini, Yu. L. Dokshitzer, G. Zanderighi  
hep-ph / 000629

# ARGUMENT OF THE COUPLING

## ONE GLUON EMISSION

$$\mathcal{W}^{(1)}(k) = \frac{N}{2} \left( \mathcal{W}_{13}^{(1)}(k) + \mathcal{W}_{23}^{(1)}(k) - \frac{1}{N^2} \mathcal{W}_{22}^{(1)}(k) \right)$$

$$\mathcal{W}_{ab}^{(1)}(k) = \frac{\alpha_s}{\pi} \frac{2P_a P_b}{(2P_a k)(2k P_b)} = \frac{\alpha_s}{\pi k_{i,ab}^2}$$

## TWO PARTON "CORRELATED" EMISSION

$$\mathcal{W}^{(2)}(k_1, k_2) = \frac{N}{2} \left( \mathcal{W}_{13}^{(2)}(k_1, k_2) + \mathcal{W}_{23}^{(2)}(k_1, k_2) - \frac{1}{N^2} \mathcal{W}_{12}^{(2)}(k_1, k_2) \right)$$

$\mathcal{W}_{ab}^{(2)}(k_1, k_2)$  may be found in

S. Catani and M. Grazzini hep-ph/9908523

## ONE GLUON + TWO SOFT PARTONS

$$\mathcal{W}_{ab}^{(2)}(k) \Rightarrow \mathcal{W}_{ab}^{(1)}(k) + \int dk_1 dk_2 \delta(k - k_1 - k_2) \mathcal{W}_{ab}^{(2)}(k_1, k_2)$$

$$\frac{\alpha_s}{\pi k_{i,ab}^2} \Rightarrow \frac{\alpha_s(k_{i,ab}^2)}{\pi k_{i,ab}^2}$$

Yu. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam

Nucl. Phys B 511 (1998) 396 hep-ph/9707532

# HARD PARTON RECOIL

## PHASE SPACE FACTORISATION

$$d\Gamma_n = \prod_{i=1}^n [dk_i] \prod_{Q=1}^3 \frac{d^3 p_a}{(2\pi)^3 2E_a} D_n \quad [dk] = \frac{d^3 k}{\pi\omega}$$

$p_a$  HARD PARTONS  $k_i$  SOFT PARTONS

$$D_n = (2\pi)^4 \delta^4 \left( \sum_{Q=1}^3 p_a + \sum_{i=1}^n k_i - Q \right) \delta^3 \left( \underline{p}_3 + \sum_{i \in R} \underline{k}_i - \underline{p}_2 \right) \delta^2 \left( \underline{p}_{2+} + \underline{p}_{2-} \Theta(\beta_{2+}) + \sum_{i \in W} \underline{k}_{i+} - \underline{p}_{2+} \right)$$

$$p_a = p_a + q_a \quad q_a \text{ RECOIL MOMENTA}$$

$$d\Gamma_n \Theta(k_{out} - \sum_a |q_{ax}| - \sum_i |k_{ix}|) = \Gamma_0 dh_n \prod_{i=1}^n [dk_i]$$

$$dh_n = d q_{2+} \prod_a d q_{ax} \Theta(k_{out} - \sum_a |q_{ax}| - \sum_i |k_{ix}|) S_n$$

$$S_n = \delta^2 \left( \underline{q}_{2+} + \sum_R \underline{k}_{i+} \right) \delta \left( q_{2x} + q_{2x}^+ + \sum_V k_{ix} \right) \delta \left( q_{2z} + q_{2z}^+ + \sum_B k_{ix} \right)$$

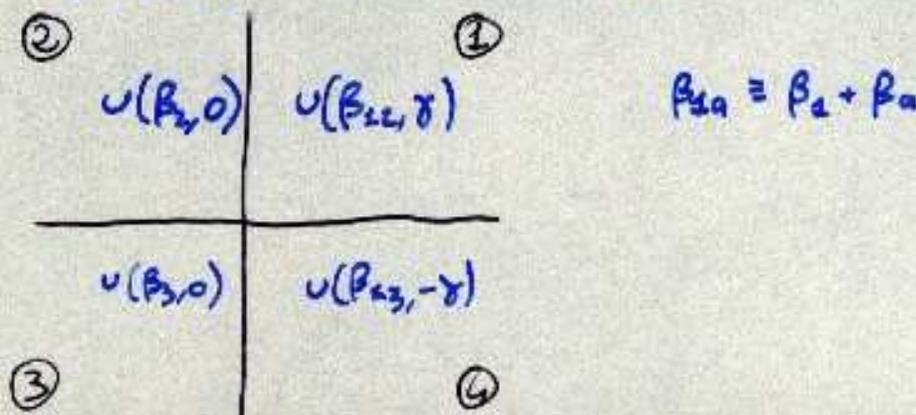
## MELLIN + FOURIER TRANSFORM $\Rightarrow$ EXPONENTIATION

$$\Sigma(k_{out}) = \int \frac{d\nu}{2\pi i \nu} e^{\nu k_{out}} \int \frac{d\gamma}{2\pi} \prod_a \frac{d\beta_a}{\pi} I(\beta_a, \gamma) e^{-E(\nu, \beta_a, \gamma)}$$

$$I(\beta_a, \gamma) = \frac{1}{1+\beta_2^2} \frac{1}{1+\beta_3^2} \left( \frac{1}{1+\beta_{2+}^2} \frac{1}{-i\gamma+E} + \frac{1}{1+\beta_{2-}^2} \frac{1}{i\gamma+E} \right)$$

$$E(\nu, \beta_a, \gamma) = \int [dk] \omega(k) \sum_{Q=1}^3 (1 - v_Q(k, \beta_a, \gamma))$$

$$v(\beta, \gamma) = \exp \{ -\nu (|k_x| + i\beta k_x + i\gamma k_y) \}$$



## RADIATOR

$$R = C_1 \int_0^{\infty} \frac{dy}{\pi(1+y^2)} r(\bar{\mu}_1, Q_1) +$$

$$C_2 r(\bar{\nu} \sqrt{1+\beta_2^2}, Q_2) + C_3 r(\bar{\nu} \sqrt{1+\beta_3^2}, Q_3)$$

where

$$r(\bar{\mu}, Q_a) = \int_{\frac{1}{\bar{\mu}}}^{\frac{Q_a}{k_x}} \frac{dk_x}{k_x} \frac{\alpha_s(2k_x)}{\pi} \log \frac{Q_a^2}{k_x^2} \quad Q_a^2 = \frac{P_{1a}^2 e^{-g_a}}{4}$$

$$\bar{\mu}_1 = (\bar{\nu} \sqrt{(1-iy)^2 + \beta_2^2}) / (\bar{\nu} \sqrt{(1+iy)^2 + \beta_3^2}) \quad \bar{\nu} = \nu e^{\gamma_E}$$

## GEOMETRY DEPENDENCE

$$P_{1a}^2 = \frac{2P_a P_b \ 2P_a P_c}{2P_b P_c}$$

- DUE TO SOFT INTER-JET GLUONS
- DEPENDS ON  $T, T_H$

## TO SL ACCURACY

$$R = R(\bar{\nu}) + r'(\bar{\nu}) \left\{ C_1 \int_0^{\infty} \frac{dy}{\pi(1+y^2)} \log \sqrt{(1-iy)^2 + \beta_2^2} \sqrt{(1+iy)^2 + \beta_3^2} \right. \\ \left. + C_2 \log \sqrt{1+\beta_2^2} + C_3 \log \sqrt{1+\beta_3^2} \right\}$$

$$R(\bar{\nu}) = \sum_a C_a r(\bar{\nu}, Q_a)$$

$$r'(\bar{\nu}) = \frac{\alpha_s(k_x)}{\pi} \log \frac{Q^2}{k_x^2} \Big|_{k_x = \frac{1}{\bar{\nu}}}$$

# DISTRIBUTION

$$\Sigma(K_{out}) = e^{-R(\bar{K}_{out}^{\pm})} \frac{F(\bar{K}_{out}^{\pm})}{\Gamma(1 + R'(\bar{K}_{out}^{\pm}))}$$

$$R_{out} = K_{out} e^{-\gamma_E}$$

$$R' = (2C_F + N) r'$$

$$F(\bar{K}_{out}^{\pm}) = \frac{r'(\bar{K}_{out}^{\pm})}{2} (2C_1 + C_2 + C_3) \log 4 + \dots$$

## SL CORRECTIONS

- IN THE RADIATOR R
  - 2) ARGUMENT OF THE COUPLING  $\alpha_s(2k_x)$
  - 3) HARD PARTON DECAY  $Q_{\alpha}$
  - 4) MOMENTUM SCALES  $Q_{\alpha} \Rightarrow T, T_n$  DEPENDENCE
- IN THE FUNCTION F
  - 1) HARD PARTON RECOIL

## COMPARISON WITH EXPERIMENTAL DATA

- ALL AVAILABLE DATA EXPLORE THE REGION  $T \lesssim 1$   
NOT INTERESTING SINCE WE LOSE THE RICHNESS OF  
3 JET TOPOLOGY  $R \simeq C_F v(\bar{v}, Q)$
- CASE FOR DATA IN THE "RESTRICTED" REGION  
 $T_n \lesssim T \lesssim 0.8$

## SOME WORK TO DO FOR US IN THE MEANTIME

- COMPARISON WITH NUMERICAL NLO COMPUTATIONS IN  
ORDER TO CHECK RESUMMATION  $O(\alpha_s^2 \log^2(K_{out}))$   
ALREADY CHECKED  $O(\alpha_s \log K_{out})$
- COEFFICIENT FUNCTION  
 $\Sigma(K_{out}) \rightarrow C(\alpha_s(Q)) \Sigma(K_{out})$   $C(\alpha_s) = 1 + C_2(T, T_n) \alpha_s(Q) + \dots$
- NP POWER CORRECTIONS  $k_x \sim \Delta_{QCD}$   
 $K_{out} \rightarrow K_{out} + \Delta K_{out}$   
 $\Delta K_{out} \sim \frac{A}{Q} \log \frac{Q_{NP}}{K_{out}}$   $A \sim \int_0^{\infty} \frac{dm^2}{m^2} m \alpha_{eff}(m^2)$

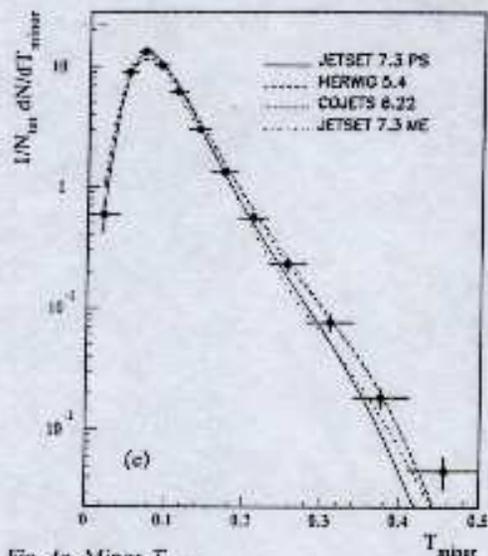
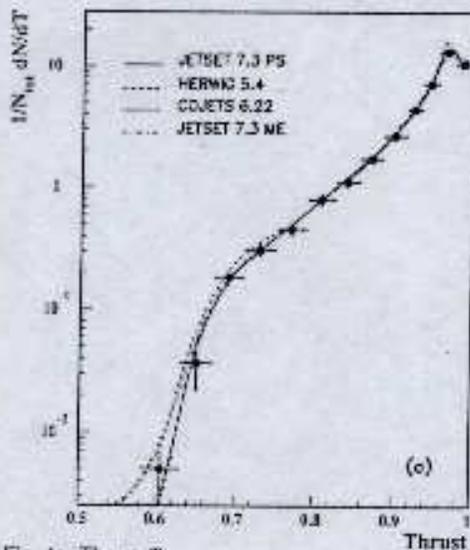
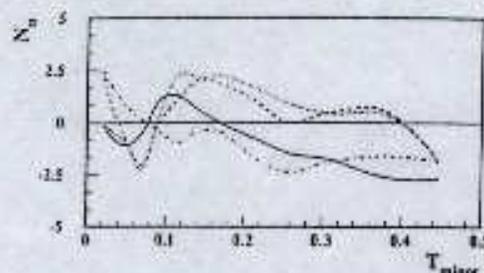
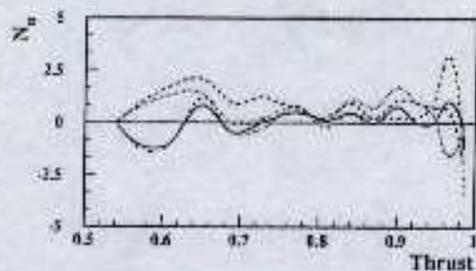


Fig. 4a. Thrust  $T$

Fig. 4c. Minor  $T_{minor}$

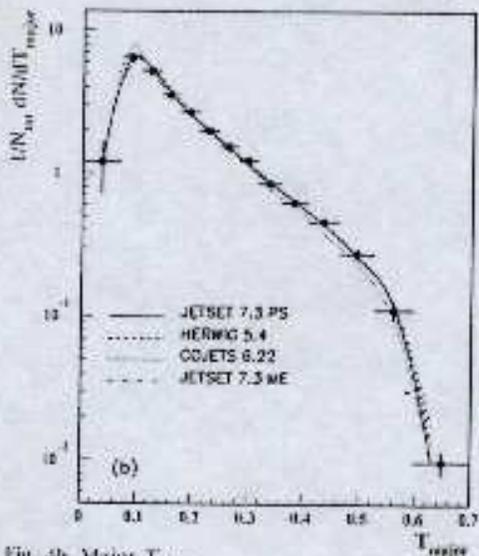
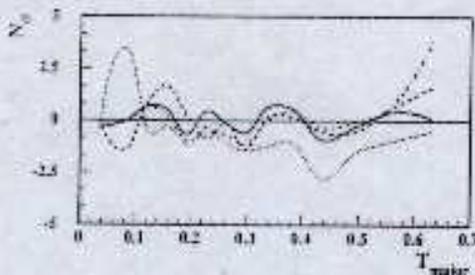
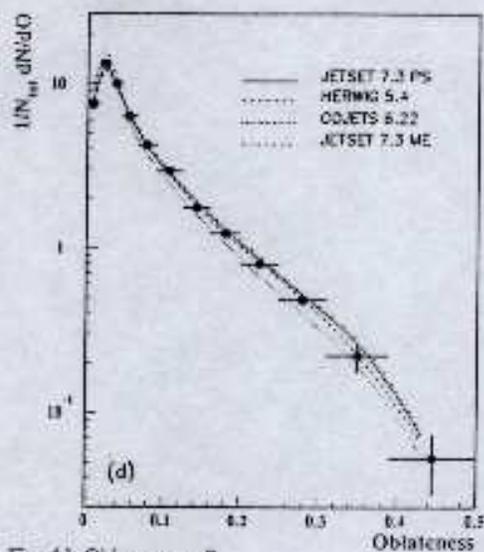
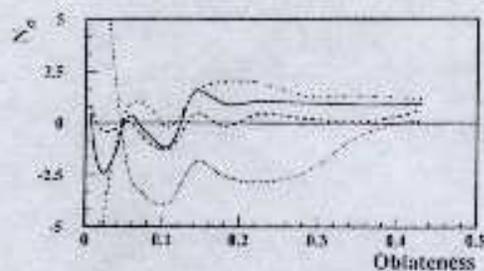
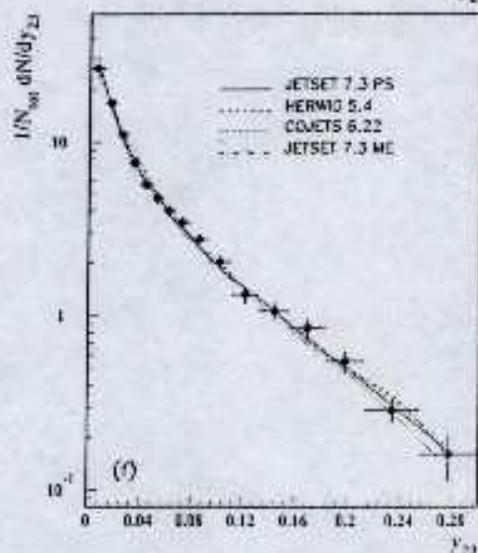
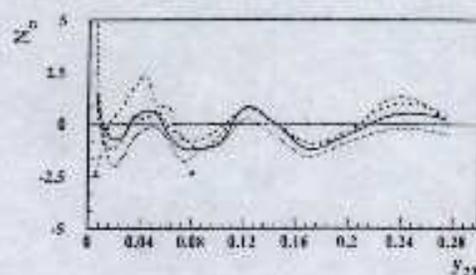
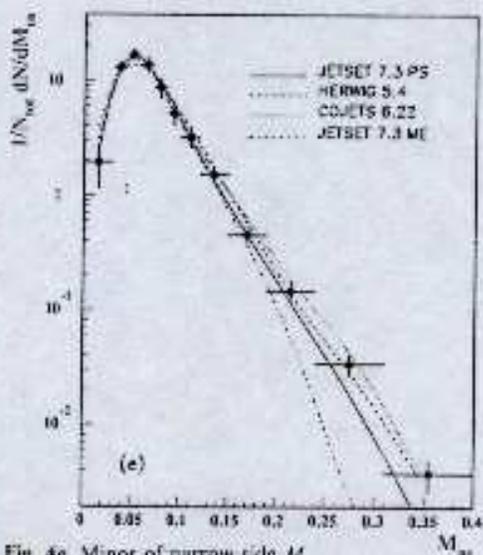
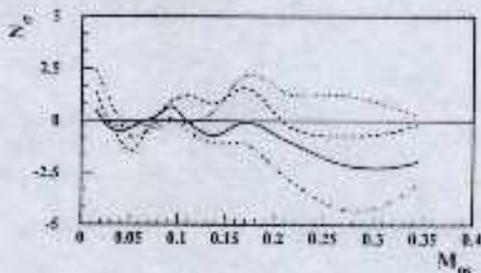
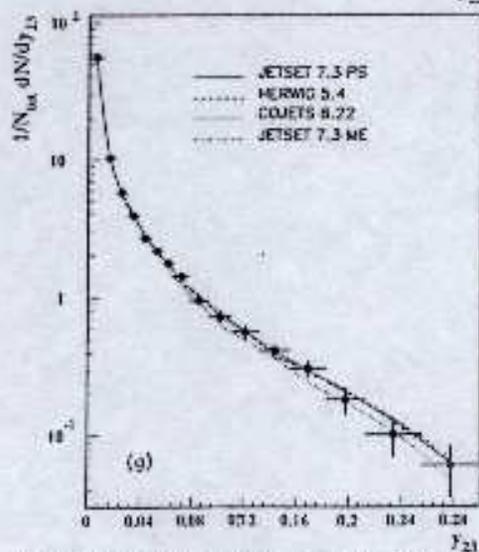
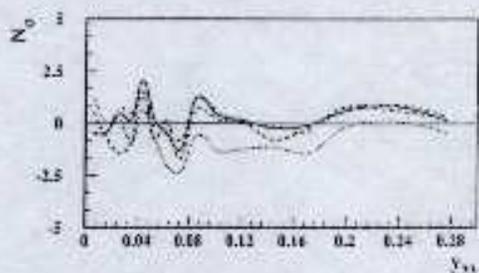


Fig. 4b. Major  $T_{major}$

Fig. 4a-p. Comparisons between the unfolded data and the predictions of the JETSET 7.3 PS, HERWIG 5.4, COJETS 6.22, and JETSET 7.3 ME Monte Carlo programs with their optimized parameter values. The dots represent the data while the lines are the predictions of the Monte Carlo models. The statistical and systematic errors are added in quadrature. The top plot shows the deviations between the data and the predictions in units of errors on the unfolded data points. All histograms are normalized to the total number of events in the sample

Fig. 4d. Oblateness  $O$ Fig. 4f. 3-jet resolution parameter of JADE algorithm  $y_{23}^{JADE}$ Fig. 4e. Minor of narrow side  $M_{10}$ Fig. 4g. 3-jet resolution parameter of  $k_t$  algorithm  $y_{23}^k$