Fulvio Piccinini
INFN – Sezione di Pavia
Dipartimento di Fisica Nucleare e Teorica
Università di Pavia

$4f(\gamma)$ e ISR a LEP2

On behalf of the Four-Fermion Working Group of the LEP2 MC Workshop

XII Convegno sulla Fisica al LEP
Trieste, 26-28 Aprile, 2000

- Higher order QED corrections to Single W
  - The issue has been addressed by two groups
    - SWAP: G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini
    - GRACE: Y. Kurihara, M. Kuroda and Y. Shimizu
  - Main problem: determination of the proper scale in the electron structure function/parton shower and the related theoretical uncertainty
- $4f$ plus a visible photon
  - Tuned comparison among several programs to fix the technical precision
  - Study of the ISR QED corrections and their theoretical uncertainty
  - Investigation of finite fermion mass effects in realistic event selections
  - Theoretical uncertainty coming from variations in the renormalization scheme

Higher order QED corrections to Single W production

In the Leading Log approximation, the factorization theorems allow to write the QED corrected cross section of a generic process as a convolution of the form

$$d\sigma = \prod_i \int dx_i D(\Lambda_i^2, x_i)\, d\sigma_0$$

The choice of the scales Λ_i is not dictated by general arguments.
A generally adopted attitude is given by the comparison of the $O(\alpha)$ expansion of the above convolution with a diagrammatic calculation which reproduces the correct LL contribution

$$d\sigma = d\sigma_0 \left(1 + \sum_i \frac{\alpha}{\pi} \log \frac{\Delta E}{E} L(\Lambda_i^2) \right)$$

Tipical simple examples are $e^+e^- \to f\bar{f}$, with $f \neq e$ and Bhabha scattering, for which an exact $O(\alpha)$ perturbative calculation exist. In the first case $\Lambda_- = \Lambda_+ = s$. Considering the radiation emitted from the incoming and outgoing electron in the t-channel contribution to Bhabha scattering the right scale turns out to be $\Lambda_- = |t|$. For the full Bhabha, where s, t and their interference are present at the same time the proper scales are given by the combination $\Lambda_- = \Lambda_+ = st/u$.

M. Greco and O. Nicrosini, *Phys. Lett. B240 (1990) 219.*

In the case of single-W production, the exact $O(\alpha)$ perturbative calculation is still lacking, so a general strategy for the evaluation of the scales $\Lambda_\pm$ is needed.

The leading (double-log) contribution to photon radiation traces back to soft and collinear photon bremsstrahlung and its virtual counterpart, and, in the case of a calorimetric measurement of the energy of the final-state (FS) particles, to hard radiation collinear to the FS particles themselves

$$d\sigma_{\mathrm{S+V}} = d\sigma_0 \log\frac{\Delta E}{E}\frac{2\alpha}{\pi}\sum_{i>j}^{n} Q_i Q_j \log\frac{s_{ij}}{m_i m_j}$$

with $s_{ij} \equiv (q_i + q_j)^2$.

Notice that in the limit $s_{ij} \ll m_i^2, m_j^2$

$$\log\frac{s_{ij}}{m_i m_j} \longrightarrow \frac{1}{3}\frac{s_{ij}}{m_i m_j}$$

Summing up the contribution of hard collinear photons to final-state charged particles, we get

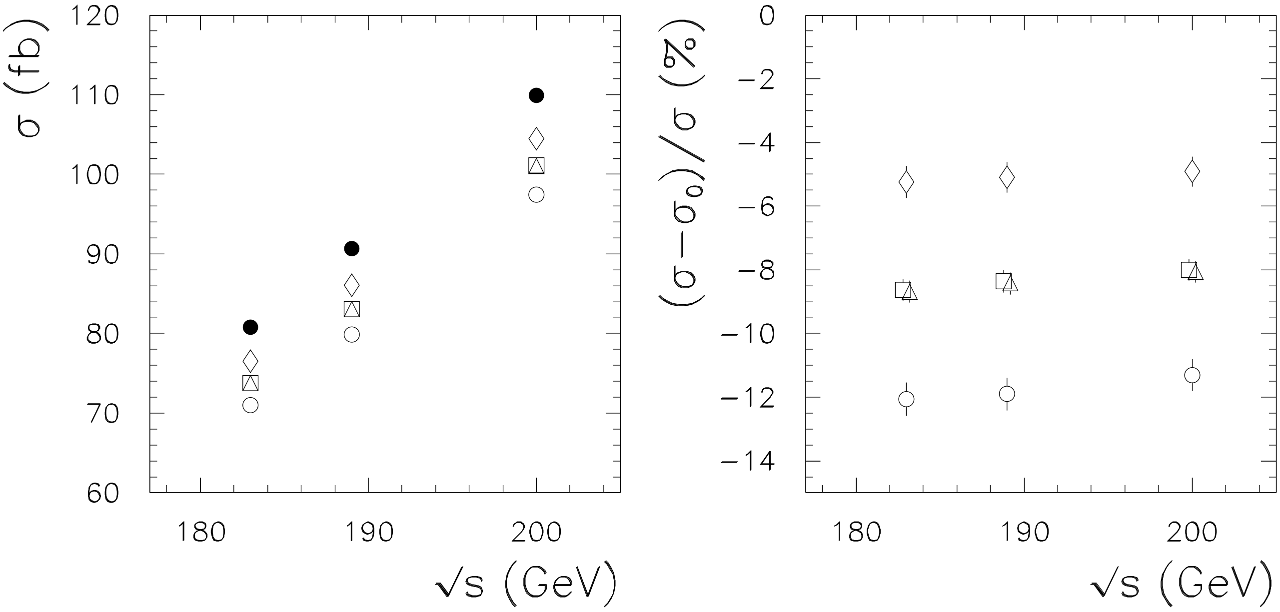
$$\begin{aligned} d\sigma_{\mathrm{S+V}} + d\sigma_{\mathrm{hard}} &= d\sigma_0 \frac{2\alpha}{\pi}\log\frac{\Delta E}{E}\left\{\sum_{i=m+1}^{n} Q_i^2 \log\frac{E_i}{m_i} + \right. \\ &\quad \left. - \sum_{i>j}^{n} Q_i Q_j \log 2(1-c_{ij}) - \sum_{i}^{m} Q_i^2 \log\delta \right\} \end{aligned}$$

The comparison between the last equation and the $O(\alpha)$ expansion of the SF QED corrected cross section allow to fix the scales Λ_i.
If a calorimetric measurement of the energies of the FS particles is performed, only the IS legs need be corrected by the SF's. Furthermore, since the electron is scattered in the very forward region, the interference between the electron line and the rest of the process is very small. This allows a natural sharing of the logarithms coming from the previous equation between the two SF's associated to the colliding electron and positron, whose scales read

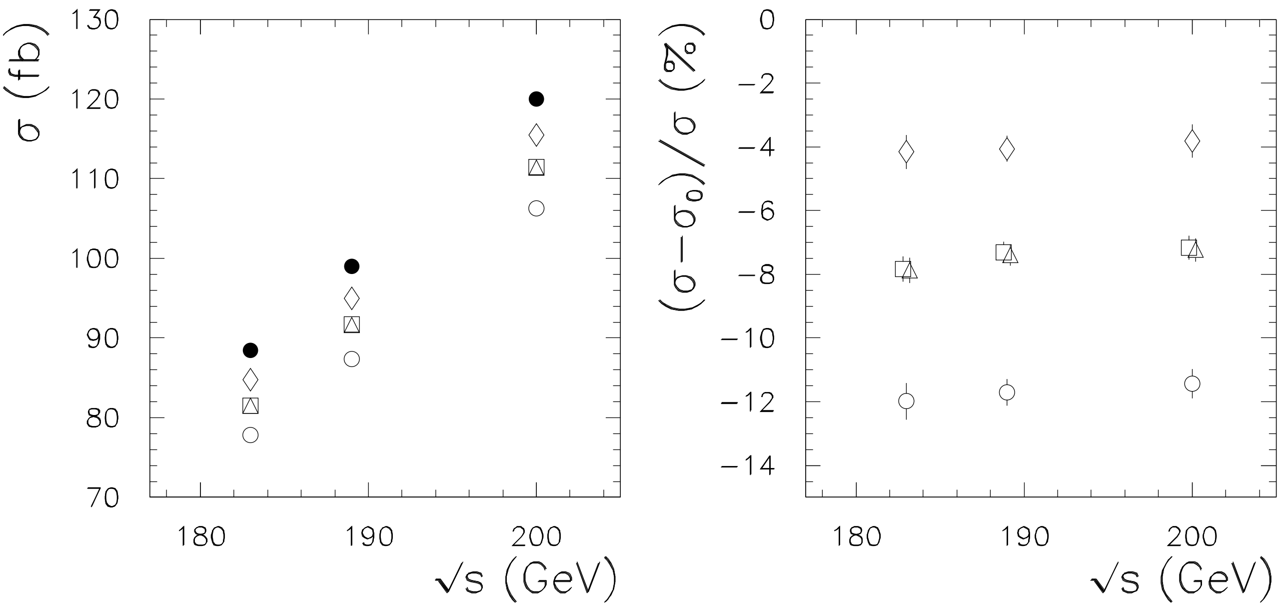
$$\Lambda_-^2 = 4E^2 \frac{(1-c_-)^2}{\delta^2}\,, \quad \Lambda_+^2 = 2^{\frac{14}{9}} E^2 \frac{\left((1-c_{\bar{d}})(1-c_u)^2\right)^{\frac{2}{3}}}{\left((1-c_{u\bar{d}})^2\delta^5\right)^{\frac{2}{9}}}$$

Naive ansatz: by thinking of the process in terms of the Weizsäcker-Williams approximation, *i.e.* in terms of a convolution of the process $e^+\gamma \to \nu_e W^*$ with an equivalent photon spectrum plus a real electron line, leads to assign two different scales to the single-W process: one scale for the electron current and one for the positron current. The former scale is the proper one for a t-channel process, so it is simply $|q^2_{\gamma^*}|$. The latter is the sum of an s-channel electron exchange and a t-channel W exchange. Assuming the t-channel dominance, its natural cut-off is given by the W-boson mass, M_W

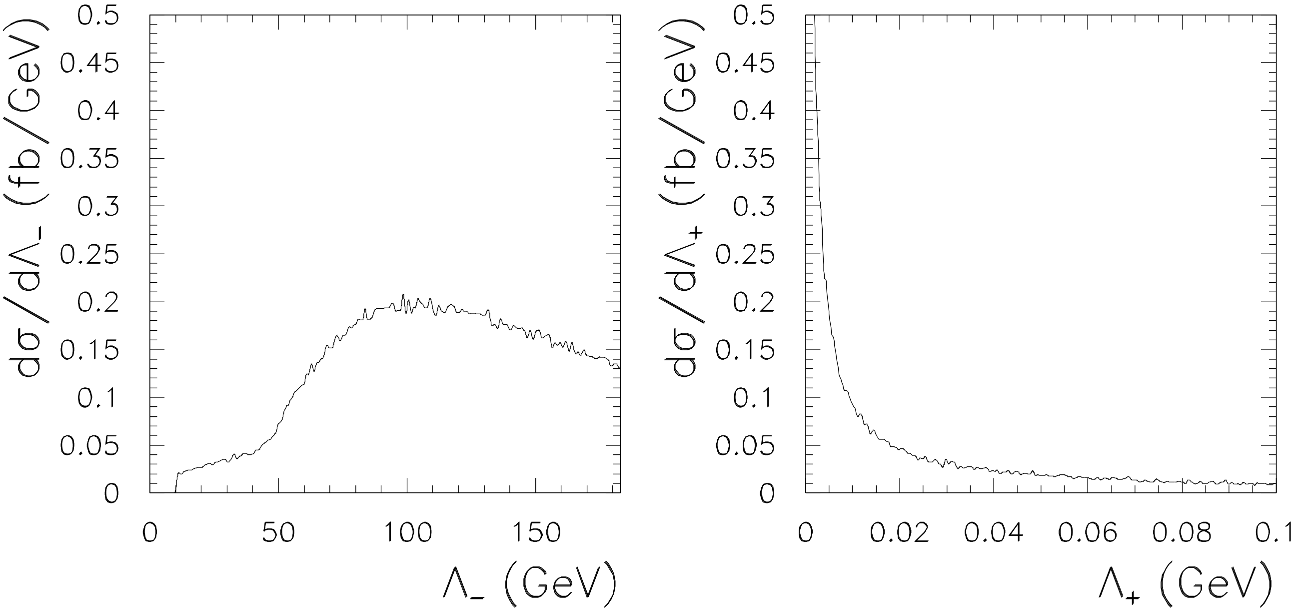
$$\Lambda^2_{-,\text{naive}} = |q^2_{\gamma^*}|\,, \quad \Lambda^2_{+,\text{naive}} = M_W^2 \tag{1}$$



The effects of LL QED corrections to the cross section of the single-W process $e^+e^- \to e^-\bar{\nu}u\bar{d}$ for different choices of the energy scale in the electron/positron SF's. The quark angular acceptance $0^\circ \leq \vartheta_{u,\bar{d}} \leq 180^\circ$ is considered. Left: absolute cross values as functions of the c.m. energy. Right: relative difference between the QED corrected cross sections and the Born one, still as functions of the c.m. energy. The marker $\bullet$ represents the Born cross section, $\bigcirc$ represents the correction given by $\Lambda^2_\pm = s$ for both SF's, $\diamond$ the correction given by the scales $\Lambda^2_\pm = |q^2_{\gamma^*}|$ for both SF's, the correction given by the naive scales, $\triangle$ the correction given by the refined scales. The entries correspond to $\sqrt{s} = 183$, 189, 200 GeV.



The same as above for the quark angular acceptance $|\cos\theta_{u,\bar{d}}| < 0.95$.



The differential cross sections of the single-W process $e^+e^- \to e^-\bar{\nu}u\bar{d}$ with respect to the two calculated scales $\Lambda_\pm$ at $\sqrt{s} = 189$ GeV.

The agreement between the predictions obtained by the naive scales and the calculated ones can be understood by looking at the above differential distributions with respect to the scales $\Lambda_\pm$

As a last remark, it should be noted that since the finite electron mass effects are crucial in the calculation of the single-W cross section, particular care has to be devoted to the kinematics after ISR: a good working procedure is to put electron and positron on their mass-shell after ISR. Otherwise they go off mass-shell and this implies a violation of the $U(1)$ Ward Identity at the level of several % (G. Passarino, hep-ph/9810416).

$4f$ plus a visible photon

Physics motivations:

- it is the only process measurable at LEP2 which involves quartic gauge couplings
- it allows the production of three gauge bosons ($WW\gamma$, $ZZ\gamma$, $Z\gamma\gamma$)
- it is an important building block of the radiative corrections to $4f$ production

The process has been extensively studied within the LEP2 MC Workshop held at CERN

Available generators:

- `CompHEP`

  E. Boos, M. Dubinin and V. Ilyn

- `GRACE`

  Y. Kurihara, M. Kuroda and Y. Shimizu

- `NEXTCALIBUR`

  F.A. Berends, C. Papadopoulos and R. Pittau

- `PHEGAS/HELAC`

  C. Papadopoulos

- `RacoonWW`

  A. Denner, S. Dittmaier, M. Roth and D. Wackeroth

- `WRAP`

  G. Montagna, M. Moretti, O Nicrosini, M. Osmo and F. Piccinini

A detailed tuned comparison on cross-sections and distributions has been performed by `RacoonWW`, `WRAP` and `PHEGAS/HELAC`

Processes considered

- $\mu\, \bar{\nu}_\mu\, u\, \bar{d}\, \gamma$
- $e^-\, \bar{\nu}_e\, u\, \bar{d}\, \gamma$
- $\mu\, \bar{\nu}_\mu\, \tau^+\, \nu_\tau\, \gamma$
- $e^-\, \bar{\nu}_e\, \tau^+\, \nu_\tau\, \gamma$
- $s\, \bar{c}\, u\, \bar{d}\, \gamma$

Observables studied at $\sqrt{s} = 200\,\mathrm{GeV}$

- integrated cross-sections;
- E_γ distribution, $d\sigma/dE$ [fb/GeV];
- $\cos\theta_\gamma$, $d\sigma/d\cos\theta_\gamma$ [fb];
- angle $\theta_{f\gamma}$ between photon and the nearest charged final-state fermion, $d\sigma/d\theta_{\gamma f}$ [fb];
- $M_{u\bar{d}}$, $M_{\tau^+\nu_\tau}$ invariant mass, $d\sigma/dM$ [fb/GeV];
- $M_{e^-\bar{\nu}_e}$, $M_{\mu^-\bar{\nu}_\mu}$, $M_{s\bar{c}}$, invariant masses, $d\sigma/dM$ [fb/GeV];

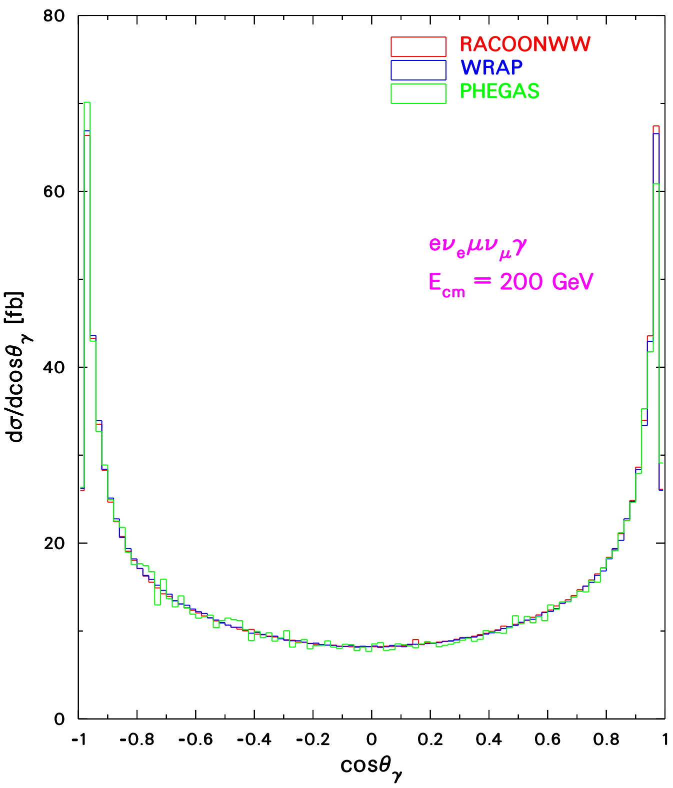
Applied cuts

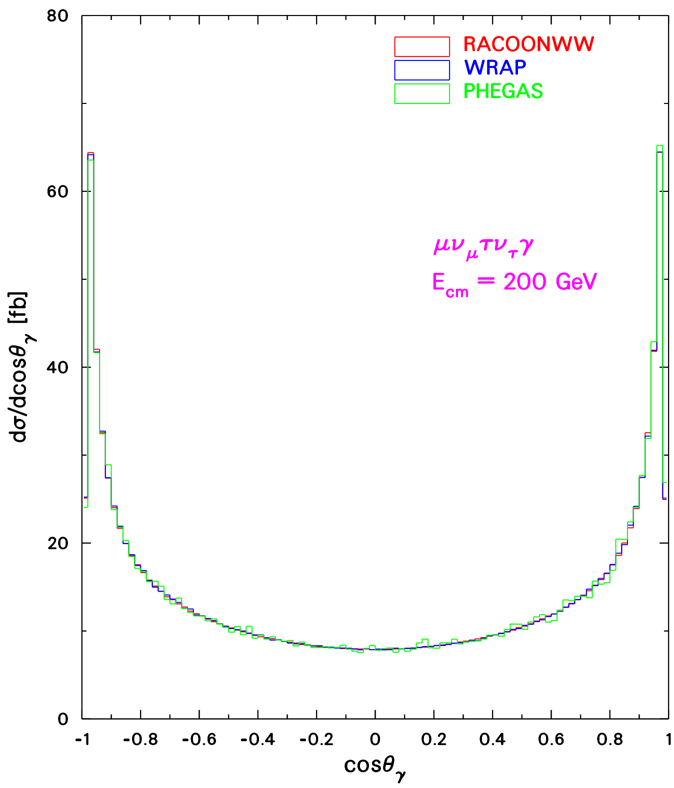
- common to all processes: $E_\gamma > 1\,\mathrm{GeV}$, $|\cos\theta(\gamma, \mathrm{beam})| < 0.985$, $\theta(\gamma, \mathrm{f}) > 5°$, $\mathrm{f} =$ charged fermion.
- for $ud\mu\nu_\mu$ and $ude\nu_e$: $M(ud) > 10\,\mathrm{GeV}$
- $M_{e^-\bar{\nu}_e}$ for the process $e^-\, \bar{\nu}_e\, \tau^+\, \nu_\tau\, \gamma$, $d\sigma/dM$ [fb/GeV];
  $|\cos\theta(\mathrm{l}, \mathrm{beam})| < 0.985$ $E_\mathrm{l} > 5\,\mathrm{GeV}$, where l is a charged lepton,
- for $\tau\nu_\tau\mu\nu_\mu$ and $\tau\nu_\tau e\nu_e$: $|\cos\theta(\mathrm{l}, \mathrm{beam})| < 0.985$, $E_\mathrm{l} > 5\,\mathrm{GeV}$, $M(l^+l^-) > 10\,\mathrm{GeV}$,
- for $udcs$: at least two pairs with $M(q_i q_j) > 10\,\mathrm{GeV}$.

Resut: good technical agreement at the 0.1% level

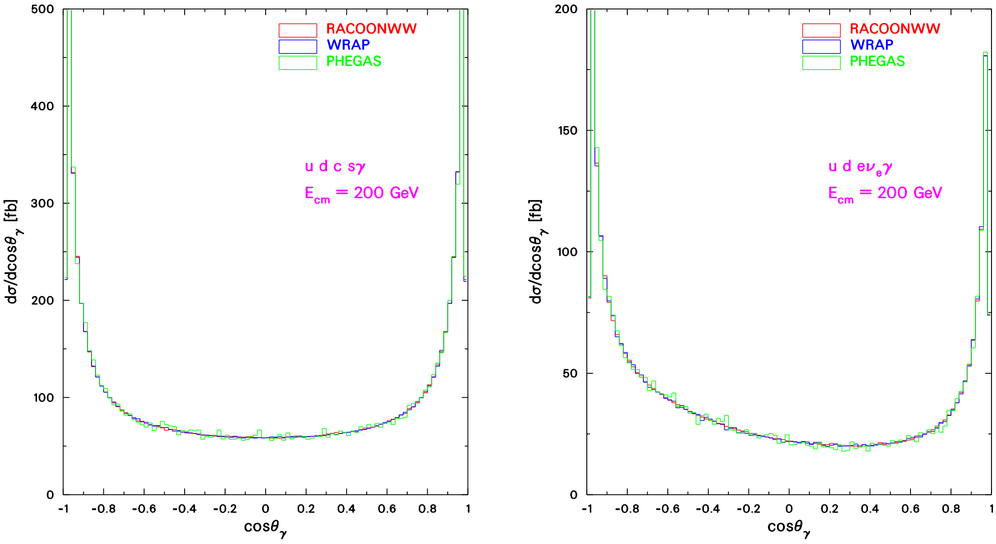
Process	WRAP	RacoonWW	PHEGAS/HELAC
$u\bar{d}\mu^-\bar{\nu}_\mu$	75.732(22)	75.647(44)	76.200(350)
$u\bar{d}e^-\bar{\nu}_e$	78.249(43)	78.224(47)	78.140(423)
$\nu_\mu\mu^+\tau^-\bar{\nu}_\tau$	28.263(9)	28.266(17)	28.359(111)
$\nu_\mu\mu^+e^-\bar{\nu}_e$	29.304(19)	29.276(17)	29.185(154)
$u\bar{d}s\bar{c}$	199.63(10)	199.60(11)	200.48(81)

Comparison between WRAP, RacoonWW and PHEGAS/HELAC for a sample of total cross-sections (fb)

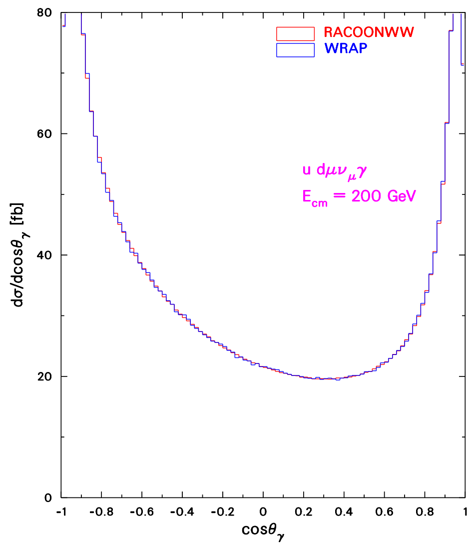




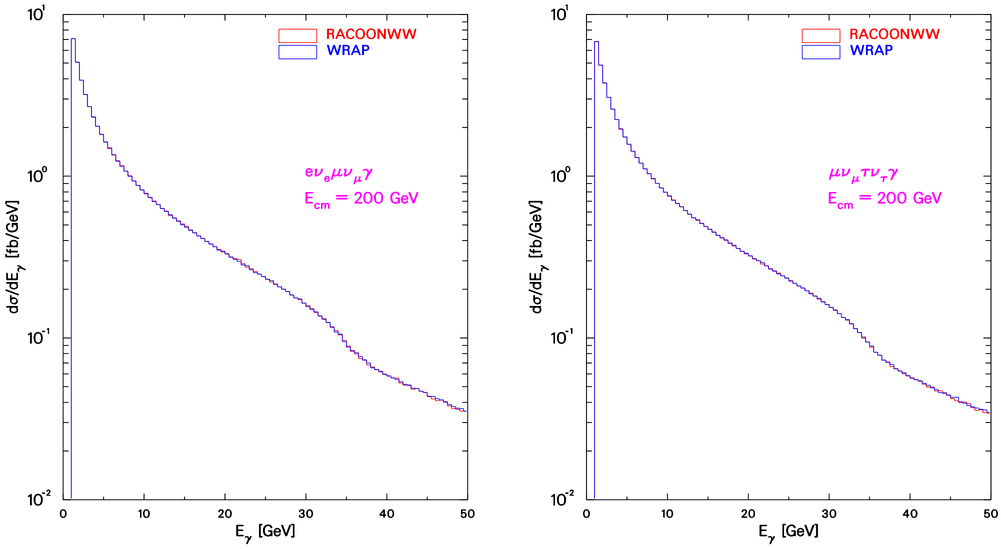
$\cos\theta_\gamma$ distribution for the processes $\nu_\mu\mu^+e^-\bar{\nu}_e\gamma$ and $\nu_\mu\mu^+\tau^-\bar{\nu}_\tau\gamma$



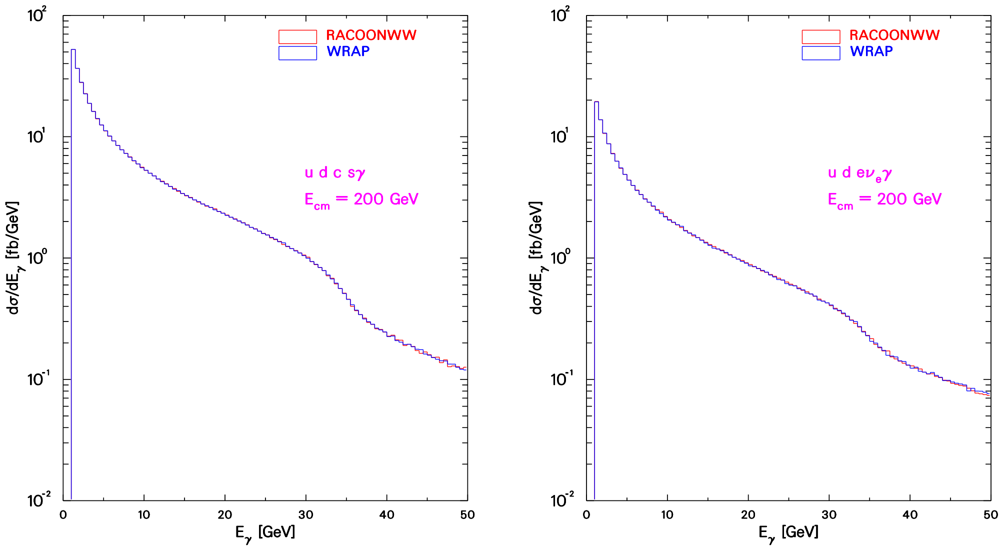
$\cos\theta_\gamma$ distribution for the processes $u\bar{d}s\bar{c}\gamma$ and $u\bar{d}e^-\bar{\nu}_e\gamma$



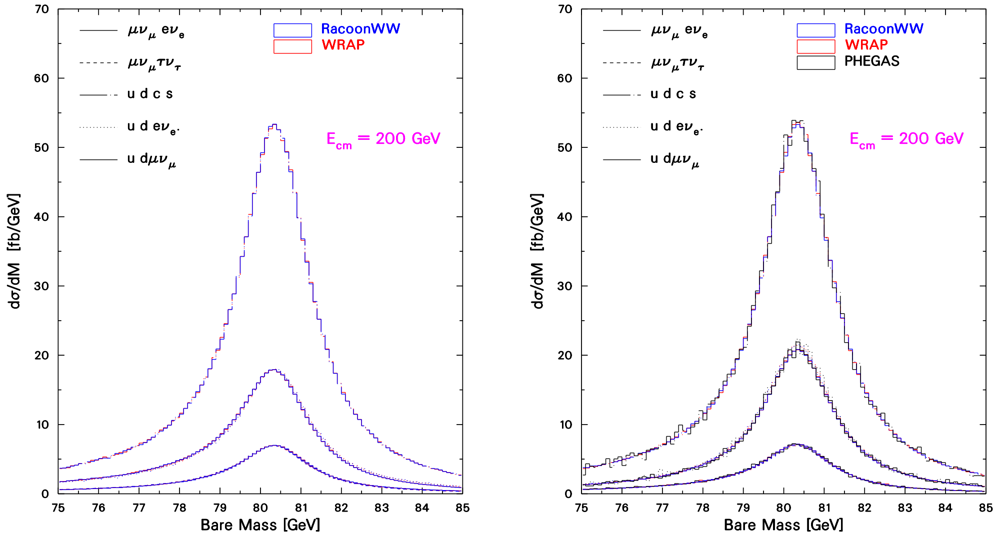
$\cos\theta_\gamma$ distribution for the process $u\bar{d}\mu^-\bar{\nu}_\mu\gamma$



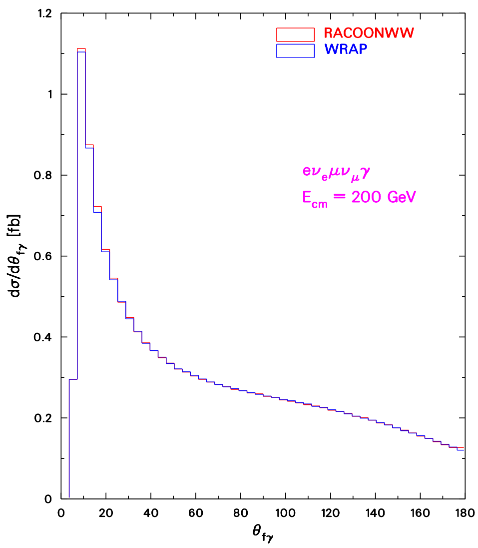
E_γ distribution for the processes $\nu_\mu \mu^+ e^- \bar{\nu}_e \gamma$ and $\nu_\mu \mu^+ \tau^- \bar{\nu}_\tau \gamma$



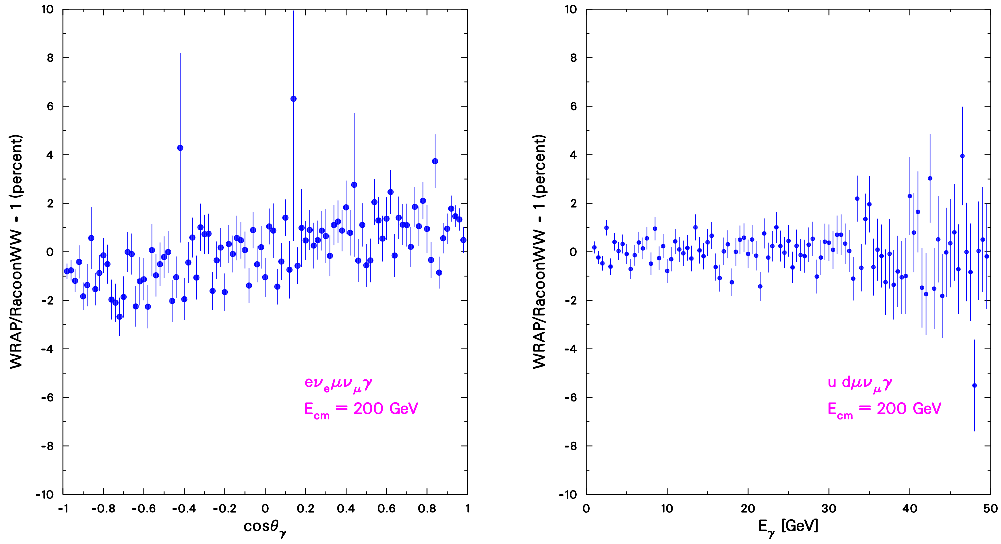
E_γ distribution for the processes $u\bar{d}s\bar{c}\gamma$ and $u\bar{d}e^-\bar{\nu}_e\gamma$



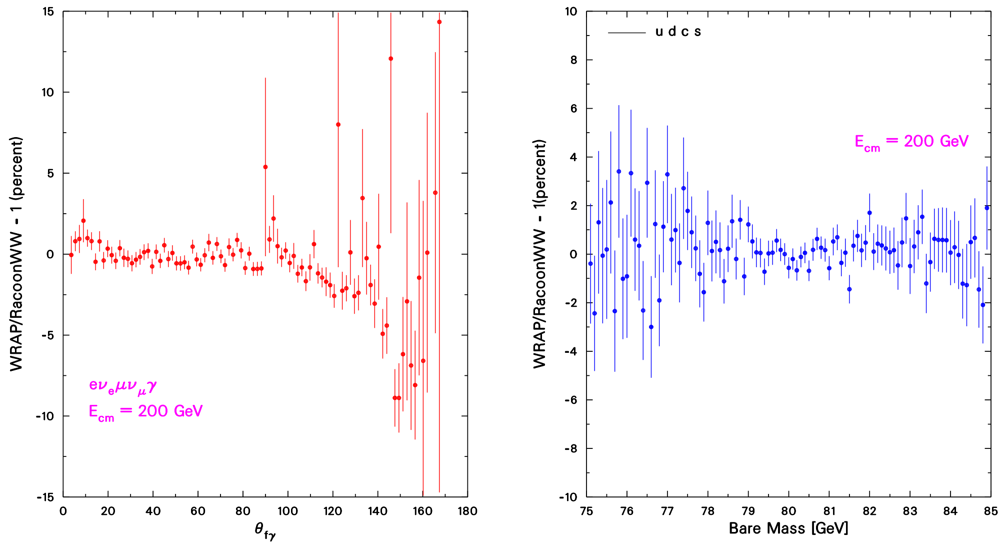
Bare W^- and W^+ mass distributions



$\theta(\gamma, \mathrm{chargedfermion})$ distribution in the process $e\nu_e\mu^-\bar{\nu}_\mu\gamma$



The ratio `WRAP/RacoonWW` for the $\cos\theta_\gamma$ distribution in the process $\nu_\mu\mu^+e^-\bar{\nu}_e\gamma$ and for the E_γ distribution in the process $u\bar{d}\mu^-\bar{\nu}_\mu\gamma$



The ratio `WRAP/RacoonW W` for the $\theta(\gamma, \text{chargedfermion})$ distribution in the process $e\nu_e\mu^-\bar{\nu}_\mu\gamma$ and for Bare W^- mass distribution in the process $u\bar{d}s\bar{c}\gamma$

Fermionic masses

The tuned comparisons have been performed by adopting the massless approximation for the outgoing fermions. However, fermionic mass terms can become important, due to the collinear "singularities" associated with photons emitted from the external legs, i.e. fermion mass effects are expected to be relevant for small angular separation cuts photon-charged fermions.

Moreover, a final state muon can be distinguished from a collinear photon. In this case the separation cut can be even 0°.

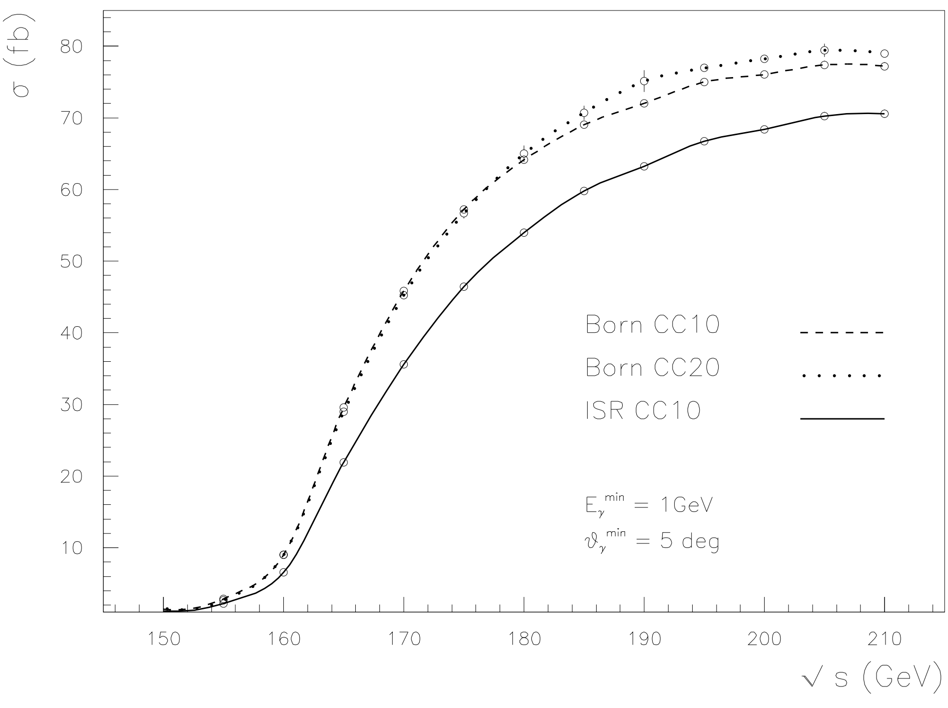
$\vartheta_{\gamma-q}$ (deg)	$\vartheta_{\gamma-\mu}$ (deg)	Cross Section (fb)	δ (%)
5°	1.0°	90.157 ± 0.036 91.903 ± 0.035	1.92 ± 0.08
5°	0.1°	104.777 ± 0.046 115.004 ± 0.044	9.31 ± 0.09
5°	0.0°	105.438 ± 0.045	

Comparison between massive and massless Born cross sections for the process $\mu^- \bar{\nu}_\mu c \bar{s} \gamma$ at $\sqrt{s} = 200$ GeV, as obtanied by means of WRAP. $\theta_{\gamma-f}$, with $f = q, \mu$ is the minimum separation angle between the photon and final state charged fermions. In the third column, the first result refers to the massive case, and the second one to the massless case. Relative difference is shown in the last column.

Initial State Radiation

The phenomenologically relevant Leading Log QED corrections due to ISR can be implemented via the Structure Functions formalism

$$\sigma_{QED}^{4f+1\gamma} = \int dx_1 dx_2\, D(x_1, s) D(x_2, s) \Theta(\text{cuts}) d\sigma^{4f+1\gamma}$$



The effect of ISR, simulated by collinear SF, on the integrated cross section of the CC10 final state $\mu^- \ \bar{\nu}_\mu \ u \ \bar{d} \ \gamma$ as a function of the LEP2 c.m. energy. The Born cross-section for the CC20 final state $e^- \ \bar{\nu}_e \ u \ \bar{d} \ \gamma$ is also shown. Numerical results by means of WRAP.

Due to the presence of an observed photon in the final state, the treatment of ISR in terms of collinear SF's can be inadequate because affected by double counting between the pre-emission photons (described by the SF) and the observed one (described by the hard-scattering matrix element). By keeping under control also the transverse degrees of freedom of ISR, as allowed by p_t-dependent SF, it is possible to remove the double-counting effects.

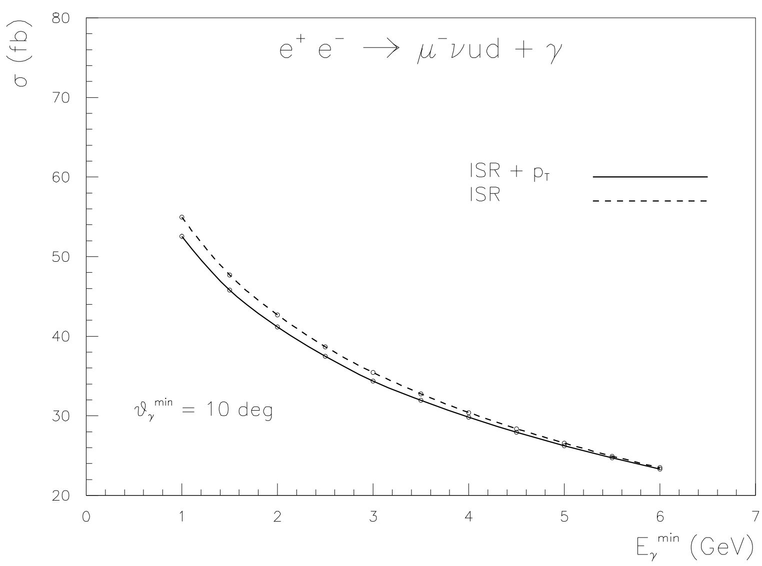
G. Montagna, O. Nicrosini and F. P., *Comp. Phys. Commun.* 98 (1996) 206

G. Montagna, M. Moretti, O. Nicrosini and F. P., *Nucl. Phys.* B541 (1999) 31

$$\sigma_{QED}^{4f+1\gamma} = \int dx_1 dx_2 dc_\gamma^{(1)} dc_\gamma^{(2)} \, \tilde{D}(x_1, c_\gamma^{(1)}; s) \tilde{D}(x_2, c_\gamma^{(2)}; s) \Theta(\text{cuts}) d\sigma^{4f+1\gamma}$$

An *equivalent* photon is generated for each colliding lepton and accepted as a higher-order ISR contribution if:

- the energy of the equivalent photon is below the threshold for the observed photon E_γ^{min}, for arbitrary angles; or
- the angle of the equivalent photon is outside the angular acceptance for the observed photons, for arbitrary energies.



Theoretical uncertainty coming from the renormalization scheme

By using WRAP two different renormalization schemes have been implemented in order to estimate their theoretical uncertainty, with the process $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ and cuts used in the tuned comparisons. The following two schemes have been adopted:

$$I) \quad s_W^2 \;=\; 1 - \frac{M_W^2}{M_Z^2}, \quad \alpha = \frac{4\sqrt{2}\,G_F M_W^2 s_W^2}{4\,\pi}, \quad g^2 = 4\,\pi\,\frac{\alpha}{s_W^2},$$

$$II) \quad s_W^2 \;=\; \frac{\pi\alpha(2M_W)}{\sqrt{2}G_F M_W^2}, \quad g^2 = 4\sqrt{2}G_F M_W^2, \quad \text{with } \alpha(2M_W^2) = 128.07$$

$\sqrt{s}$ [GeV]	cross section [fb]	δ
200 (I) 200 (II)	75.750(29) fb 75.887(29) fb	0.18%
189 (I) 189 (II)	71.889(25) fb 71.997(25) fb	0.15%
183 (I) 183 (II)	67.238(22) fb 67.324(22) fb	0.13%

Conclusions

- Higher order QED corrections to Single W
  - The issue has been studied independently by two groups (`SWAP` and `GRACE`) with similar results
  - A general procedure relying on the soft and collinear approximation of the radiative process has been investigated to determine the proper scales in the IS QED structure functions
  - By simply fixing both scales to s or t would imply a theoretical error of the order of 4%
- $4f$ plus a visible photon
  - Tuned comparison among several programs fix the technical precision at the 0.1% level or better
  - ISR QED corrections are sizable and require particular care, due to the presence of a detected photon in the final state. The use of QED collinear structure functions can determine a theoretical uncertainty of several % for realistic event selections
  - Finite fermion mass effects are important for small separation angle between photons and charged final state particles. This effect is enhanced in the case of a final state muon, where the separation angle can become 0°
  - The theoretical uncertainty coming from variations in the renormalization scheme is small compared to the previous effects, i.e. at the 0.1% level